EFFICIENT METHOD FOR PROBABILISTIC FIRE SAFETY ENGINEERING

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ABSTRACT

A growing interest exists within the fire safety community for the topics of risk and reliability. However, due to the high computational requirements of most calculation models, traditional Monte Carlo methods are in general too time consuming for practical applications. In this paper a computationally very efficient methodology is for the first time applied to structural fire safety. The methodology allows estimating the probability density function which describes the uncertain response of the fire exposed structure or structural member, while requiring only a very limited number of model evaluations. The application of the method to structural fire safety is illustrated by two examples in the area of concrete elements exposed to fire.

1 INTRODUCTION

The advent of Performance Based Design (PBD) for fire safety engineering is resulting in an increased interest of the fire safety community in the concepts of risk and reliability. Applying these concepts for structural fire safety requires evaluating the uncertain response of the structure or structural element during fire exposure. Traditionally this is done using Monte Carlo simulations (MCS), see for example Eamon and Jensen[1] or Sidibé et al.[2]. However, considering that the Monte Carlo methodology requires a very large number of model realizations, the method proves impractical for many applications of structural fire safety engineering. Computationally more efficient methods exist – see for example Guo and Jeffers[3], or Van Coile et al.[4] –, but these can be difficult to implement, and/or require prior knowledge on the type of probability density function describing the uncertain structural response. While for ambient design situations a lognormal distribution is generally considered to be an appropriate assumption for describing the uncertain strength of structural members[5], the nonlinear effects introduced by fire exposure tend to severely undermine this assumption, as has been illustrated for the bending moment capacity of concrete slabs[6]. In summary, the application of risk- and reliability-based concepts to (structural) fire safety engineering is currently severely hampered by the lack of a computationally efficient methodology which is capable of approximating an unbiased estimation of the full PDF describing the uncertain response.

2 THE BASICS – WHY EVALUATING THE PDF IS NECESSARY

For calculation models in fire safety engineering, any model output Y can be considered as a function of a number of input variables x_i. Evaluating for example the maximum temperature T_{max} in a fire engulfed compartment, T_{max} will be a function of amongst others the fire load density, the compartment dimensions, and the opening factor. Some variables may be well known and can consequently be modelled by a single deterministic value. In the model for T_{max}, this will in general be the case for the compartment dimensions. Other variables may be less clearly defined and should be modelled as stochastic variables. In the example above, the fire load density and – possibly – the opening factor fall in the latter category. By considering the uncertainty on the input variables x_i, the model output Y will be uncertain as well. Denoting with x the vector of stochastic variables and h the modelled relationship, the discussion above is mathematically written as:

$$Y = h(\mathbf{x}) \quad [1]$$

Considering a failure criterion for the output variable Y, i.e. failure if $Y > y_{limit}$, the probability of failure P_f can be calculated from the PDF f_y describing Y, through:

$$P_f = \int_{y_{limit}}^{\infty} f_y(y)\,dy \quad [2]$$

The calculated probability of failure should subsequently be compared with an acceptability criterion. For example, for structural failure in ambient design conditions, EN 1990[7] specifies an acceptability limit for P_f of $\Phi(-3.8) = 7.23 \cdot 10^{-5}$ for structures with normal failure consequences and a 50 year reference period. Here Φ denotes the standard cumulative normal distribution function.

Note that the probabilistic calculations and an explicit evaluation of the failure probability are often avoided by considering characteristic values for the uncertain input variables, for example considering a 90% quantile for the fire load density. While this procedure seems very similar to the use of characteristic values in traditional prescriptive design calculations, their application is fundamentally different as prescribed characteristic values are (or should be) based on underlying full-probabilistic calculations of P_f and a comparison with an (implicit) acceptability criterion. In other words: when applying prescriptive design rules the achievement of an acceptably low probability of failure can be assumed to result from the combination of characteristic values, safety factors and conservative assumptions, but this does not hold for innovative performance-based design solutions. For true reliability-based design solutions an explicit evaluation of P_f is required, and therefore the PDF f_y has to be determined.

As mentioned crude Monte Carlo simulations can be applied to make a direct "empirical" evaluation of P_f, effectively by-passing the need to evaluate the PDF f_y. However, for most practical applications the computational requirements for Monte Carlo simulations are too high.

3 MAXIMUM ENTROPY MULTIPLICATIVE DIMENSIONAL REDUCTION METHOD

Recently, a computationally very efficient method has been developed by Zhang[8]. The method is known as the (Fractional-Moment) Maximum Entropy Multiplicative Dimensional Reduction Method (ME-MDRM) and has been successfully applied to computationally very demanding structural Finite-Element calculations by Balomenos et al.[9]. At it's very core, the method makes an unbiased estimation of the probability density function describing the uncertain output by using the criterion of maximum entropy on a set of calculated model outputs or observed test results. The calculation concept for the maximum-entropy estimation is considered to be the mathematically correct procedure for avoiding biases with respect to the unknown PDF type or shape[10]. Furthermore, Novi Inverardi and Tagliani[11] and Zhang[8] propose the use of fractional moments for the maximum-entropy calculation, since these fractional moments are found to result in more stable estimates. The computational requirements of the method are reduced by considering Gaussian interpolation, instead of crude Monte Carlo simulations, for calculating the aforementioned fractional moments. A further reduction of computational requirements is obtained by considering multiplicative dimensional reduction. As will be shown below, the different aspects of the method fit very nicely together as for example the multiplicative dimensional reduction method allows for a very easy (approximated) calculation of the fractional moments. The above description may seem daunting and challenging, but the detailed description below shows that the actual mathematical difficulty is not too high, and the performance of the method is excellent, as illustrated by the application examples at the end of this paper, even for the difficult and nonlinear case of fire exposure. Furthermore, the method can very easily be applied together with existing programs and models. For a standard application of the method, the total number of model evaluations required for application of the method equals $nL+1$, with n the number of stochastic parameters and L the number of Gauss integration points. Consequently, in case of 5 stochastic parameters and 5 Gauss integration points, only 26 model evaluations are required, compared to thousands of model evaluations required for the application of traditional Monte Carlo Methods. However, the required ME-MDRM model evaluations can be further reduced to n (L-1) + 1, as shown in the next section.

4 THE CALCULATION METHODOLOGY

At the core of the methodology is the estimation of the PDF describing the stochastic output variable Y based on the principle of maximum entropy. As shown by Novi Inverardi and Tagliani[11], this principle results in an estimated PDF given by equation [3], with m the estimation order, λ_i estimated coefficients and α_i estimated exponents. The coefficient λ_0 normalizes the PDF – i.e. ensures that the integral of the estimated PDF across the entire domain of Y equals 1 – and is given by equation [4]. The optimum values for the exponents α_i and coefficients λ_i are determined by minimizing the (Kullback-Leibler) divergence between the true PDF and the estimated PDF[8]. Mathematically elaborating this results in the minimization criterion of equation [5], with $M^{\alpha_i}{}_Y$ the $\alpha_i{}^{th}$ sample moment of Y. For a (random) set of N realizations y_k this sample moment is given by equation [6].

$$\hat{f}_Y(y) = \exp\left(-\lambda_0 - \sum_{i=1}^{m} \lambda_i y^{\alpha_i}\right) \qquad [3]$$

$$\lambda_0 = \ln\left[\int_Y \exp\left(-\sum_{i=1}^{m} \lambda_i y^{\alpha_i}\right) dy\right] \qquad [4]$$

$$Minimize:\ \lambda_0 + \sum_{i=1}^{m} \lambda_i M_Y^{\alpha_i} \qquad [5]$$

$$M_Y^{\alpha_i} = \frac{\sum_{k=1}^{N} y_k^{\alpha_i}}{N} \qquad [6]$$

Determining the set of exponents and coefficients which minimizes [5] can be realized through readily available optimization algorithms and results in a mathematical formulation for the estimated PDF in equation [3]. For many optimization algorithms the solution may however be highly sensitive to the algorithm starting solution. As confirmed by Tagliani in personal correspondence, it is recommendable to perform a large number of independent optimizations by considering a Monte Carlo simulation for this starting solution. As the computational requirements are centred around the evaluation of the (computationally expensive) model describing Y, this reliance on a crude Monte Carlo (or Latin Hypercube Sampling) for the optimization procedure does not constitute a problem and is computationally relatively inexpensive.

In principle the estimation order m can be freely chosen, but while a higher estimation order will result in a better agreement with the input data, a too high estimation order may introduce spurious relationships for (unavoidably) limited sets of input data y_k. In general, the use of a third or fourth order (m = 3 or 4) has been proved sufficient for the estimation of the PDF[8,9]. In order to increase efficiency, the exponents α_i can be limited without loss of generality to real numbers in the range [-2; 2].

In the discussion above, the evaluation of the sample moment $M^{\alpha_i}{}_Y$ has not been elaborated in detail. As indicated by equation [6], the sample moment can in principle be determined through a crude Monte Carlo simulation, but this would severely undermine the goal of avoiding the need to perform many computationally expensive evaluations of the model describing Y. This problem is alleviated by considering multiplicative dimensional reduction in conjunction with Gaussian interpolation.

Applying multiplicative dimensional reduction, equation [1] is conceptually approximated by equation [7], with h_0 the model response when all n stochastic input variables are set equal to their median values $\hat{\mu}$, and h_l the unidimensional cut functions defined by equation [8]. The unidimensional cut functions effectively isolate the effect of the different stochastic input variables, and thus result in an approximation when combined to consider to overall model response $h(x)$.

$$Y = h(\mathbf{x}) \approx h_0^{1-n} \prod_{l=1}^{n} h_l(x_l) \quad [7]$$

$$h_l(x_l) = h(\hat{\mu}_1, ..., \hat{\mu}_{l-1}, x_l, \hat{\mu}_{l+1}, ..., \hat{\mu}_n) \quad [8]$$

Considering this multiplicative dimensional reduction and considering the different stochastic variables x_l to be independent, the k^{th} moment of the stochastic model response Y is given by [9], with $E[.]$ the expectance operator.

$$E\left[Y^k\right] = E\left[\left(h(\mathbf{x})\right)^k\right] \approx h_0^{k(1-n)} \prod_{l=1}^{n} E\left[\left(h_l(x_l)\right)^k\right] = h_0^{k(1-n)} \prod_{l=1}^{n} \int_{x_l} \left(h_l(x_l)\right)^k f_{x_l}(x_l) dx_l \quad [9]$$

The evaluation of the k^{th} moment for the i^{th} cut function is approximated with great accuracy by considering Gaussian quadrature. In its most basic form, Gaussian quadrature approximates the integration of a function $g(z)$ over the entire domain of a standard normally distributed variable Z by a weighted sum of a limited number of well-chosen evaluation points z_j, as mathematically given by equation [10] with ϕ the standard normal PDF, L the number of Gauss integration points (for most cases 5 integration points is sufficiently accurate), and w_j the associated Gauss weights[8]. For $L = 5$ the Gauss points z_j and associated weights w_j are given in Table 1.

$$\int_{-\infty}^{\infty} g(z)\phi(z) dz \approx \sum_{j=1}^{L} w_j g(z_j) \quad [10]$$

Table 1. Gauss integration points and associated Gauss weights for $L = 5$

j	z_j	w_j
1	-2.85697	0.011257
2	-1.35563	0.222076
3	0	0.533333
4	1.35563	0.222076
5	2.85697	0.011257

Equation [6] can be generalized to [11] for non-standard normal distributed variables X, with F_x^{-1} the inverse cumulative density function of X.

$$\int_x g(x) f_x(x) dx = \int_{-\infty}^{\infty} g\left(F_x^{-1}(\Phi(z))\right)\phi(z) dz \approx \sum_{j=1}^{L} w_j g\left(F_x^{-1}\left(\Phi(z_j)\right)\right) \quad [11]$$

Considering the k^{th} power of the cut function h_l and the probability density functions f_{xl} as specific situations of equation [11], the application of Gaussian quadrature for the evaluation of equation [9] is straightforward, resulting in an approximation for the moment $M_Y^{\alpha i}$ by:

$$M_Y^{\alpha_i} \approx \left[h_0^{1-n}\right]^{\alpha_i} \prod_{l=1}^{n} \sum_{j=1}^{L} w_j \left[h_l\left(F_{x_l}^{-1}\left(\Phi(z_j)\right)\right)\right]^{\alpha_i} = \left[h_0^{1-n}\right]^{\alpha_i} \prod_{l=1}^{n} \sum_{j=1}^{L} w_j y_{j,l}^{\alpha_i} \quad [12]$$

with $y_{j,l}$ the model realization for the j^{th} Gauss point in the l^{th} cut function, as mathematically given by:

$$h_l\left(F_{x_l}^{-1}\left(\Phi(z_j)\right)\right) = y_{j,l} \quad [13]$$

In summary, equation [12] replaces equation [6]. Consequently, the estimation of the full PDF describing Y is obtained by considering 1 model evaluation for h_0 and nL model evaluations for the cut functions. Note that calculating a different power α_i in the minimization procedure of equation [5] does not require new model evaluations. Furthermore, if the number of Gauss integration points L is uneven, one of the Gauss points z_j equals 0, resulting in one of the Gauss points equal to the median . This further reduces the number of required model calculations to $1 + n\cdot(L\text{-}1)$. Consequently, when considering 5 Gauss integration points, the total number of model evaluations required for approximating any power of the output variable Y is 1 + 4n.

5 EXAMPLE 1: SIMPLE MATHEMATICAL FORMULA

Before focussing on applications in the field of fire safety engineering, it is beneficial to illustrate the application of the methodology by a mathematical example which can easily be recalculated. Consider equation [14], with X_1, X_2 and X_3 three independent lognormal variables. Given mean values equal to 3, 4 and 2, and coefficients of variation of 0.3, 0.5 and 0.2 for X_1, X_2 and X_3 respectively, the stochastic output variable Y is mathematically defined by a lognormal distribution as well, with a mean value of 12.48 and a coefficient of variation of 0.646.

$$Y = 2\frac{X_1 X_2}{X_3} \qquad [14]$$

While the PDF describing Y is mathematically known, the ME-MDRM methodology can be applied as well, considering only the required $1 + 3\cdot(5\text{-}1) = 13$ model evaluations (for $L = 5$). These evaluations and the underlying values for z_j are given in Table 2, with $z_{j,l}$ the Gaussian point for variable X_l, $x_{j,l}$ the corresponding realization for X_l, and $y_{j,l}$ the model evaluation given through equation [14]. For clarity, the row with all $z_{j,l} = 0$ has been repeated in Table 2 for each of the cut function evaluations. Naturally, this repeating identical set of input variables has to be evaluated only once. Having evaluated the model for each of the Gauss point combinations, any moment $M_Y^{\alpha i}$ of Y is readily approximated through equation [12]. Consequently, the optimization of equation [5] can be applied, resulting in values for the coefficients λ_i and exponents α_i.

Table 2. Gaussian points and corresponding evaluations of equation [14]

$z_{j,1}$	$z_{j,2}$	$z_{j,3}$	$x_{j,1}$	$x_{j,2}$	$x_{j,3}$	$y_{j,l}$
0	0	0	2.8735	3.5777	1.9612	10.4841
-2.85697	0	0	1.2421	3.5777	1.9612	4.5320
-1.35563	0	0	1.9301	3.5777	1.9612	7.0420
0	0	0	2.8735	3.5777	1.9612	10.4841
1.35563	0	0	4.2780	3.5777	1.9612	15.6085
-2.85697	0	0	6.6473	3.5777	1.9612	24.2532
0	-2.85697	0	2.8735	0.9279	1.9612	2.7190
0	-1.35563	0	2.8735	1.8858	1.9612	5.5261
0	0	0	2.8735	3.5777	1.9612	10.4841
0	1.35563	0	2.8735	6.7876	1.9612	19.8902
0	-2.85697	0	2.8735	13.7949	1.9612	40.4244
0	0	-2.85697	2.8735	3.5777	1.1138	18.4609
0	0	-1.35563	2.8735	3.5777	1.4994	13.7128
0	0	0	2.8735	3.5777	1.9612	10.4841
0	0	1.35563	2.8735	3.5777	2.5651	8.0156
0	0	-2.85697	2.8735	3.5777	3.4533	5.9540

The optimum Monte Carlo optimization result for $m = 3$ is given in Table 3, defining the mathematical formulation of the estimated PDF through equation [3]. A comparison of the analytical lognormal PDF and CDF with 10000 crude Monte Carlo simulations on the one hand, and the result of the ME-MDRM calculation on the other hand is given in Figure 1 and Figure 2 – note that the horizontal axis in Figure 2 has been chosen in a more limited range in order to accentuate the differences. Both graphs very clearly demonstrate the excellent performance of the method for this simple mathematical example.

Table 3. Estimated PDF distribution parameters λ_i and α_i, for m = 3, based on 1000 Monte Carlo simulations for the optimization algorithm starting solution.

i	λ_i	α_i
0	-0.1572	0
1	8.1586	-0.7363
2	0.2030	1.7970
3	-0.1621	1.8466

Figure 1. PDF for Y: Analytical result and ME-MDRM result. Comparison with histogram of 10000 MCS.

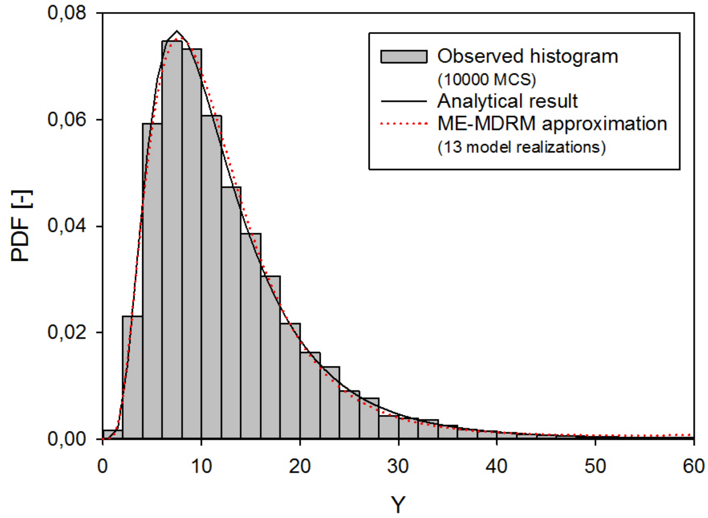
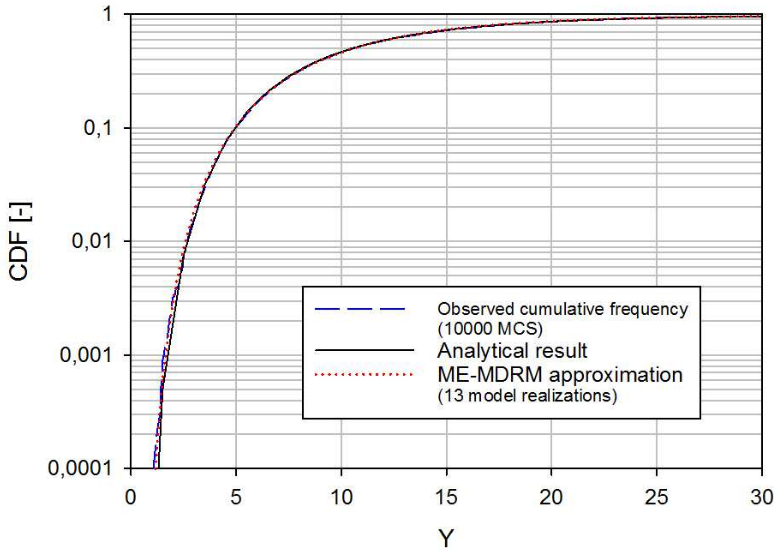


Figure 2. CDF for Y: Analytical result and ME-MDRM result. Comparison with observed cumulative frequency of 10000 MCS.



6 EXAMPLE 2: MAXIMUM VERTICAL LOAD ON A CONCRETE COLUMN

Columns in medium-rise and high-rise buildings are crucial for structural integrity and their structural fire resistance is generally considered of much higher importance than the fire resistance of for example secondary beams. In order to evaluate the risk- and reliability-performance of current prescriptive design rules and to allow for a true reliability-based structural fire design of concrete columns, the reliability of eccentrically loaded columns during fire exposure must be determined. Up to now a number of interesting Monte Carlo-based studies on the reliability of fire-exposed concrete columns do exist, see for example Sidibé et al.[2] and Achenbach and Morgenthal[12], but the computational requirements of a true second-order calculation are very high. Approximating the second-order calculation is possible, but the nonlinear effects associated with fire exposure can make these approximations difficult. Furthermore, if local exposure of a multi-storey column should be evaluated for different fire scenarios using a Finite Element model, the Monte Carlo approach becomes untenable.

A numerical model for an iterative second-order calculation of fire-exposed concrete columns has been developed by Wang et al.[13]. Here another approach is used – i.e. application of a Direct Stiffness Method (DSM) matrix calculation[14] – but for a given vertical load and eccentricity the converged results correspond with those of the model by Wang et al.

The DSM model calculates the maximum vertical load P_{max} on a fire-exposed concrete column, for a fixed eccentricity e. While less demanding than a true Finite Element model, a Monte Carlo simulation of the DSM model is still computationally very expensive. Considering the column characteristics and the 6 stochastic variables given in Table 4, and a 60 minute ISO 834 standard fire exposure (considering prescriptive requirements), application of both 10000 MCS and the ME-MDRM method (25 model evaluations, m = 4) results in the PDF and CDF for P_{max} compared in Figure 3 and Figure 4.

As illustrated by Figure 3, the ME-MDRM estimated PDF captures the behaviour of the observed histogram very well, while a lognormal approximation is found to be less appropriate (although using all MCS to estimate its parameters). Note that the lognormal approximation underestimates the occurrence of low P_{max}. These observations are confirmed in Figure 4 where it is also found that the ME-MDRM does overestimate the occurrence of very low P_{max} compared to the MCS. Note however that 1) a single MCS cannot be expected to give a precise estimation for the CDF values in the range of (1 / number of MCS), 2) that only 25 model evaluations where needed for the ME-MDRM result, 3) that the ME-MDRM is very accurate for CDF values of an order of magnitude of for example $5 \cdot 10^{-3}$. This implies that for example a characteristic value of P_{max} associated with a 99.5% confidence level is very accurately predicted by the ME-MDRM.

Table 4. Deterministic parameters and stochastic variables for the concrete column

Variable name and symbol	*Dimension*	*Distribution*	*Mean μ*	*Coefficient of variation V*
20°C concrete compressive strength f_c (f_{ck} = 55 MPa)	MPa	Lognormal	78.6	0.15
20°C reinforcement yield stress f_y (f_{ck} = 55 MPa)	MPa	Lognormal	581.4	0.07
Concrete cover c	mm	Beta μ[1-3V; 1+3V]	25	0.2
Compressive strength reduction factor at elevated temperature k_{fc}	-	Beta μ[1-3V; 1+3V]	nominal value EN 1992-1-2	temperature dependent*
Yield stress reduction factor at elevated temperature k_{fy}	-	Beta μ[1-3V; 1+3V]	nominal value EN 1992-1-2	temperature dependent*
Reinforcement area A_s (4Ø32mm)	mm^2	Normal	3217	0.02
Column width z	mm	Deterministic	300	-
Vertical load eccentricity	mm	Deterministic	5	-
Column height	m	Deterministic	4	-

* as given in (Van Coile et al., 2013)

7 EXAMPLE 3: BENDING MOMENT CAPACITY OF A CONCRETE SLAB

Evaluating the fire resistance of a concrete slab may seem like a simple problem. In agreement with EN 1992-1-2 (CEN, 2004) a specified structural fire resistance time R can be assumed as soon as the bottom reinforcement has an adequate (tabulated) axis distance from the exposed surface. It is however unclear which safety level corresponds with the accepted design solutions of

EN 1992-1-2, and knowledge of this safety level would be most beneficial when evaluating the equivalence of less standard designs (for example when externally bonded FRP (Fiber Reinforced Polymer) reinforcement has been used in a refurbishment of an existing building). Furthermore, in a more detailed PBD, the obtained safety level can be compared against explicit performance criteria agreed upon by the stakeholders.

In order to evaluate the safety level, MCS can be applied or the PDF of (for example) the bending moment capacity $M_{R,fi,t}$ at t minutes of fire exposure has to be determined. While traditionally a lognormal distribution would be assumed for the PDF describing $M_{R,fi,t}$, this assumption has been shown to result in an unsafe approximation[6]. Considering the importance of the concrete cover c for the bending capacity $M_{R,fi,t}$ a mixed-lognormal approximation should be used. However, this very specific type of PDF could only be determined as part of a research project and through a large number of MCS. It is therefore most interesting to evaluate how the ME-MDRM performs here – i.e. to assess whether the ME-MDRM is capable of identifying the irregularity of the PDF.

Figure 3. ME-MDRM result for PDF describing P_{max} at 60 minutes ISO834 for e = 0.05 m, and comparison with histogram of 10000 MCS, and a lognormal approximation (with parameters based on the MCS).

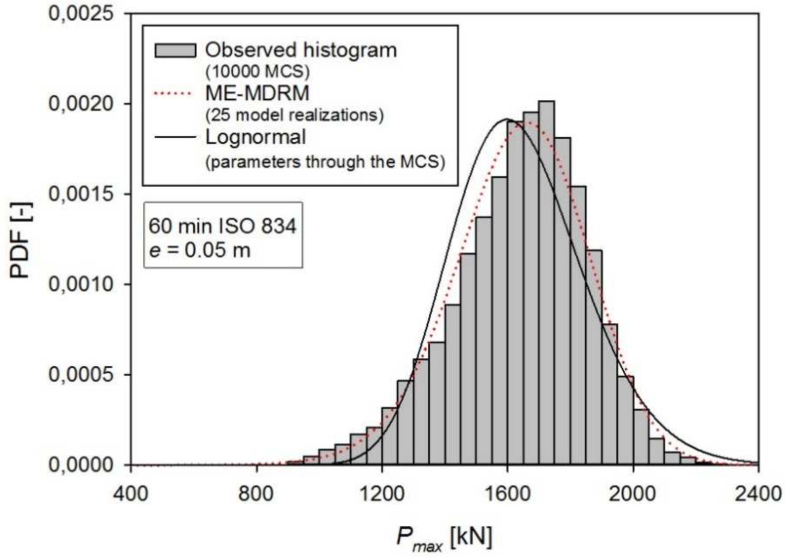
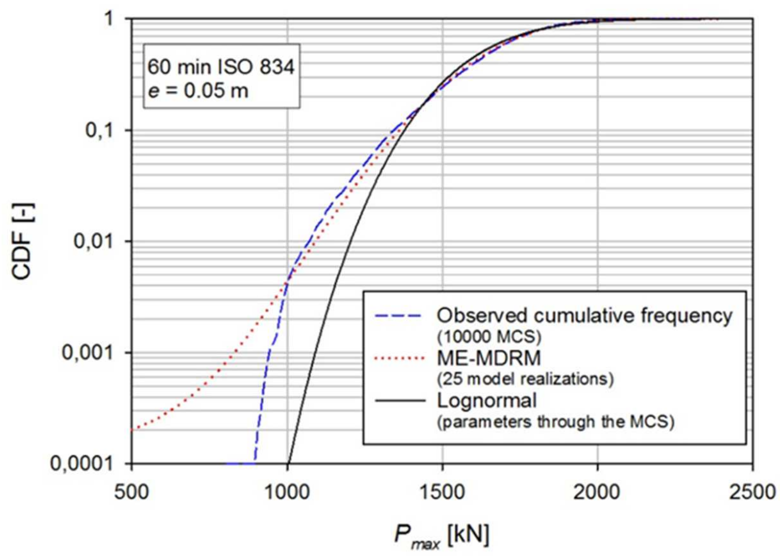


Figure 4. ME-MDRM result for CDF describing P_{max} at 60 minutes ISO834 for e = 0.05 m, and comparison with histogram of 10000 MCS, and a lognormal approximation (with parameters based on the MCS).



Consider the concrete slab configuration of Table 5. MCS for the bending moment capacity $M_{R,fi,t}$ are executed for exposure to 240 minutes of ISO 834 standard fire, using the approximate analytical model, of equation [15][6]. As Table 5 indicates 5 stochastic variables, only 21 model evaluations are needed for application of the ME-MDRM. The obtained PDF and CDF are compared in Figure 5 and Figure 6 with the mixed-lognormal approximation and the histogram of the MCS.

As shown in the Figures, the ME-MDRM results in a very reasonable approximation and correctly identifies the irregular shape of the PDF. The irregularity of the PDF estimated with ME-MDRM indicates to the user that more detailed analyses may be required. Ideally these additional analyses identify the cause of the irregularity and provide the user with additional information to consider for a reframing of the problem, see for example the reframing resulting in the proposal for a mixed-lognormal distribution[6]. Furthermore, note that the estimated PDF has an excellent agreement with the observed cumulative frequency of the MCS up to a CDF precision of 10^{-2}, indicating that for example a characteristic value with 99% confidence level is very accurately predicted.

$$M_{R,fi,t} = A_s k_{fy} f_y \left(h - c - \frac{\varnothing}{2} \right) - 0.5 \frac{\left(A_s k_{fy} f_y \right)}{b f_c} \qquad [15]$$

Table 5. Deterministic parameters and stochastic variables for the slab cross-section

Variable name and symbol	*Dimension*	*Distribution*	*Mean μ*	*Coefficient of variation V*
20°C concrete compressive strength f_c (f_{ck} = 30 MPa)	MPa	Lognormal	42.9	0.15
20°C reinforcement yield stress f_y (f_{yk} = 500 MPa)	MPa	Lognormal	581.4	0.07
Concrete cover c	mm	Beta [μ+3V; μ-3V]	35	0.14 (σ_c = 5 mm)
Yield stress reduction factor at elevated temperature k_{fy}	-	Beta [μ+3V; μ-3V]	nominal value EN 1992-1-2	temperature dependent*
Reinforcement area A_s (10Ø10mm per unit width)	mm^2	Normal	785	0.02
Slab thickness h	mm	Deterministic	200	-
Reinforcement bar diameter Ø	mm	Deterministic	10	-
Slab unit width b	mm	Deterministic	1000	-

* as given in (Van Coile et al., 2013)

Figure 5. ME-MDRM result for PDF describing $M_{R,fi,t}$ at 240 minutes ISO834, and comparison with histogram of 10000 MCS, and the mixed-lognormal approximation.

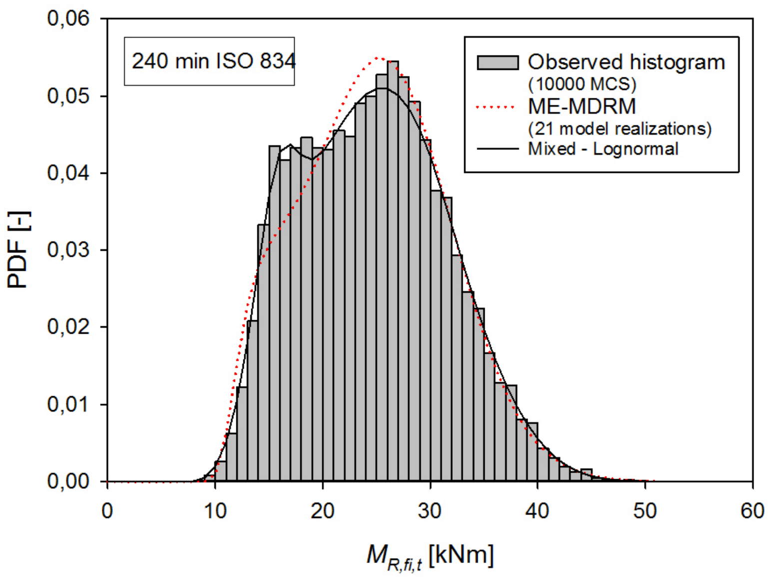
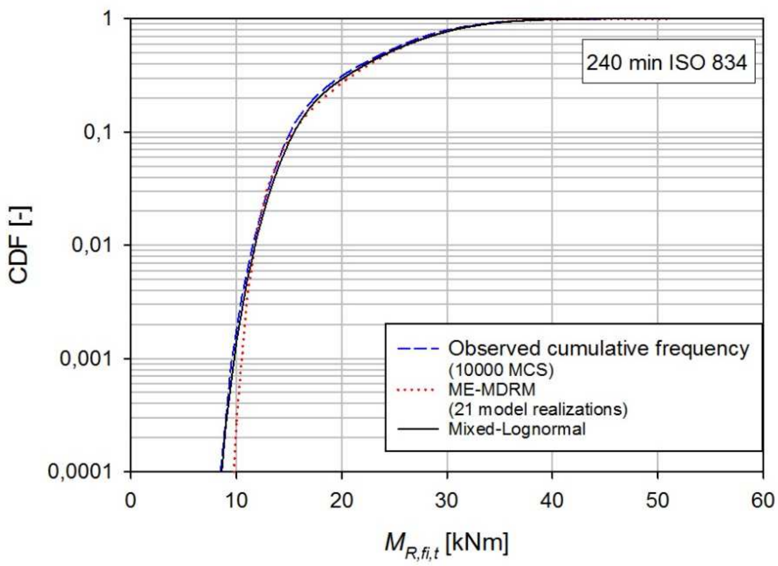


Figure 6. ME-MDRM result for CDF describing $M_{R,fi,t}$ at 240 minutes ISO834, and comparison with histogram of 10000 MCS, and the mixed-lognormal approximation.



8 CONCLUSIONS

The application of risk- and reliability concepts to structural fire safety has been significantly hampered by the computational requirements associated with Monte Carlo simulations. In order to break this impasse and to allow for probabilistic fire safety calculations of for example the global structural response in case of a localized fire, an easy-to-implement and computationally efficient methodology is required. The Maximum Entropy Multiplicative Dimensional Reduction Method (ME-MDRM) presented here may prove to have both the computational efficiency and excellent compatibility with existing models and calculation tools to fill this gap and to allow for a breakthrough in the area of risk- and reliability-based structural fire safety. The ME-MDRM results in a mathematical formula for the probability density function (PDF) describing the uncertain output variable, while requiring only a very limited number of model evaluations. The example applications above (maximum eccentric vertical load on a concrete column, and bending moment capacity of a concrete slab) illustrate the excellent performance of the method for structural fire engineering.

ACKNOWLEDGEMENT

The authors wish to thank prof. Zhang and prof. Tagliani for their fruitful comments on the application of the maximum entropy principle.

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